

Higher-order corrections to Z branching ratios

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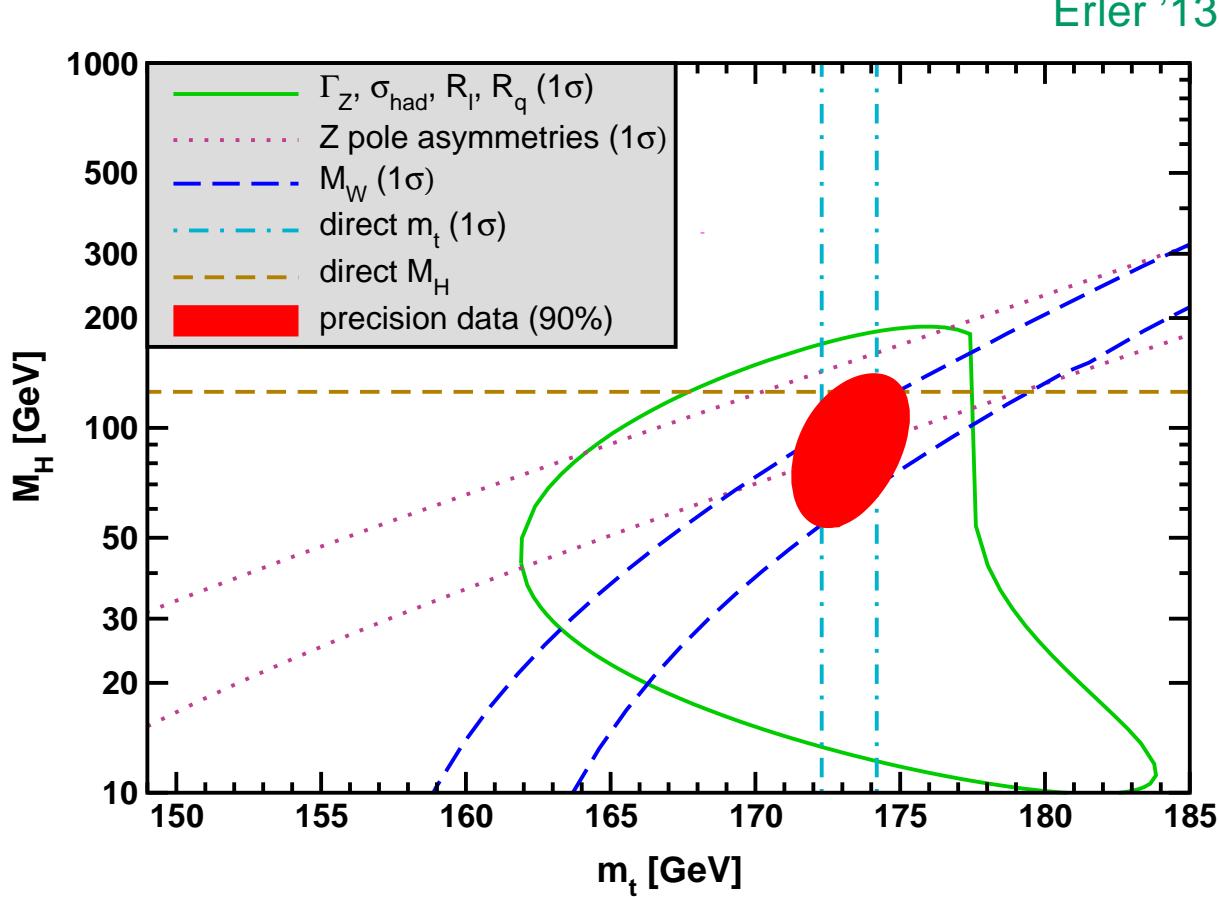
based on arXiv:1310.2256, 1401.2447

- 1. Micro-review of electroweak precision observables**
- 2. Z-boson width at 2-loop**
- 3. Results and error estimate**
- 4. Outlook**

Electroweak precision observables

Standard Model after Higgs discovery:

- Very good agreement over large number of observables
- Sensitivity to TeV-scale new physics



Direct measurements:

$$M_H = 125.6 \pm 0.4 \text{ GeV}$$

$$m_t = 173.24 \pm 0.95 \text{ GeV}$$

Indirect prediction:

$$M_H = 123.7 \pm 2.3 \text{ GeV}$$

(with LHC BRs)

$$M_H = 89^{+22}_{-18} \text{ GeV}$$

(w/o LHC data)

$$m_t = 177.0 \pm 2.1 \text{ GeV}$$

Electroweak precision observables

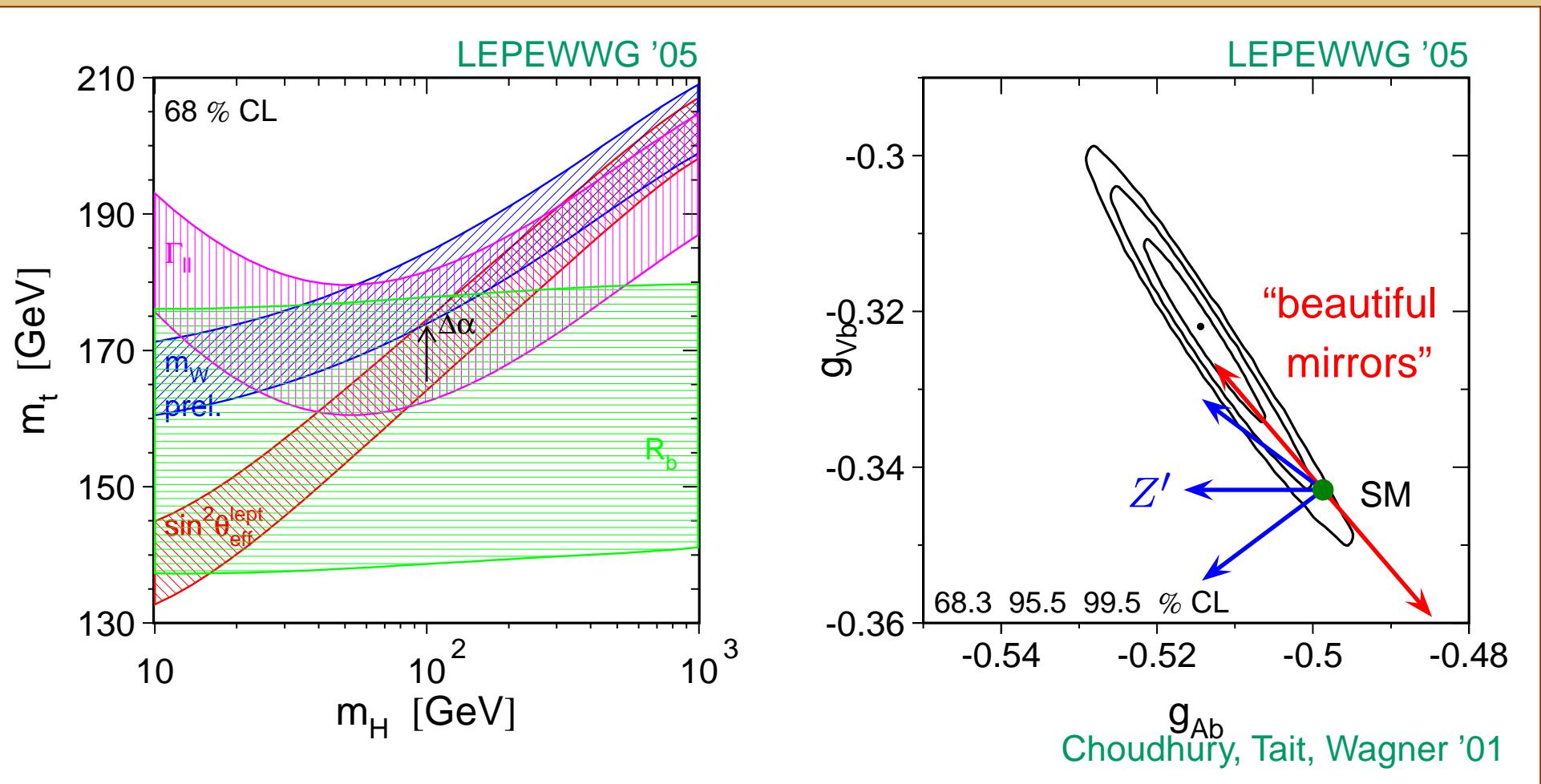
Impact of different observables:

M_W (from G_μ)

$R_b = \Gamma[Z \rightarrow b\bar{b}]/\Gamma[Z \rightarrow \text{had.}]$

$\Gamma_{ll} = \Gamma_Z \text{BR}[Z \rightarrow ll]$

$\sin^2 \theta_{\text{eff}}^\ell$ (from A_{LR} and A_{FB})



Electroweak precision observables

Most important quantities:

	Exp. error	Th. error
M_W	15 MeV	4 MeV
Γ_Z	2.3 MeV	0.5 MeV
$R_b = \Gamma[Z \rightarrow b\bar{b}] / \Gamma[Z \rightarrow \text{had.}]$	6.6×10^{-4}	1.5×10^{-4}
$\sin^2 \theta_{\text{eff}}^\ell$ (from A_{LR} and A_{FB})	16×10^{-5}	5×10^{-5}

Experimental precision requires inclusion of

- 2-loop corrections of $\mathcal{O}(\alpha\alpha_s)$ and $\mathcal{O}(\alpha^2)$
- Leading 3/4-loop corrections enhanced by m_t^n

Electroweak precision observables

Most important quantities:

	Exp. error	Th. error	ILC/GigaZ
M_W	15 MeV	4 MeV	3–5 MeV
Γ_Z	2.3 MeV	0.5 MeV	~ 1 MeV
$R_b = \Gamma[Z \rightarrow b\bar{b}]/\Gamma[Z \rightarrow \text{had.}]$	6.6×10^{-4}	1.5×10^{-4}	1.5×10^{-4}
$\sin^2 \theta_{\text{eff}}^\ell$ (from A_{LR} and A_{FB})	16×10^{-5}	5×10^{-5}	1.3×10^{-5}

ILC (TLEP) precision will require

- Dominant 3-loop corrections, with (several) closed fermion loops
- Precision of theory prediction may ultimately limited by parametric error from $\Delta\alpha$ and M_Z

Z-pole observables

After deconvolution of initial-state QED radiation and subtraction of γ -exchange:

$$\mathcal{A}[e^+ e^- \rightarrow f\bar{f}] = \frac{R}{s - s_0} + S + (s - s_0)S'$$

$$s_0 \equiv M_Z^2 - iM_Z\Gamma_Z$$

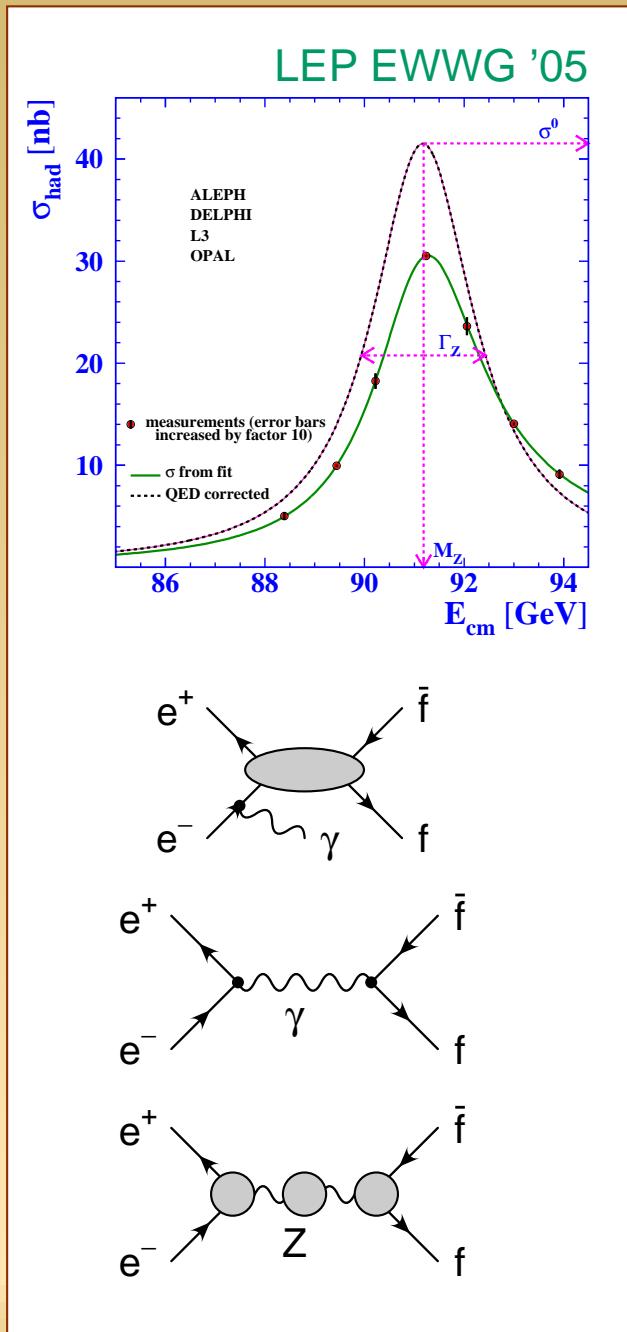
s_0, R, S, S' are gauge-invariant

Willenbrock, Valencia '91; Sirlin '91; Stuart '91
Gambino, Grassi '00

$$\sigma \sim \frac{1}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2} + \text{non.res.}$$

$$M_Z = M_Z^{\text{exp}} - 34 \text{ MeV}$$

$$\Gamma_Z = \Gamma_Z^{\text{exp}} - 0.9 \text{ MeV}$$



Z-boson width at 2-loop

Z width:

$$\begin{aligned}\Gamma_Z &= \frac{1}{M_Z} \text{Im } \Sigma_Z(s_0) \\ &= \frac{1}{M_Z} \left[\frac{\text{Im } \Sigma_Z}{1 + \text{Re } \Sigma'_Z} - \frac{1}{2} M_Z \Gamma_Z (\text{Im } \Sigma_Z) (\text{Im } \Sigma''_Z) \right]_{s=M_Z^2} + \mathcal{O}(\Gamma_Z^3).\end{aligned}$$

Optical theorem: $(Z \rightarrow Z) \Rightarrow |Z \rightarrow f\bar{f}|^2$

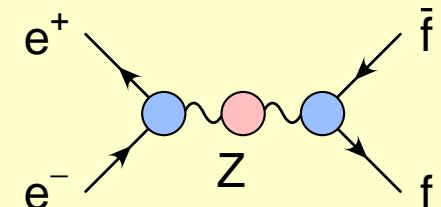
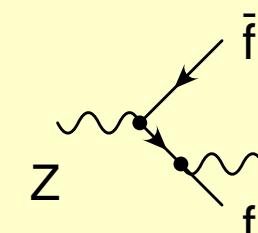
$$\Gamma_Z = \sum_f \Gamma_f, \quad \Gamma_f \approx \frac{N_c M_Z}{12\pi} \left[\left(\mathcal{R}_V^f |v_f|^2 + \mathcal{R}_A^f |a_f|^2 \right) \frac{1}{1 + \text{Re } \Sigma'_Z} \right]_{s=M_Z^2}$$

$\mathcal{R}_V^f, \mathcal{R}_A^f$: Final-state QED/QCD radiation;
known to $\mathcal{O}(\alpha_s^4), \mathcal{O}(\alpha^2), \mathcal{O}(\alpha\alpha_s)$

Kataev '92

Chetyrkin, Kühn, Kwiatkowski '96
Baikov, Chetyrkin, Kühn, Rittinger '12

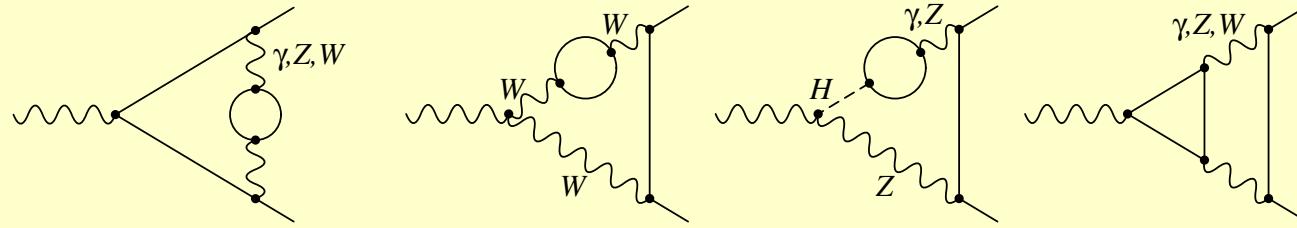
v_f, a_f, Σ'_Z : Electroweak corrections



Two-loop Z -vertex corrections

Dominant contributions:

Diagrams with closed fermion loops (enhanced by N_f and/or m_t^2)



■ Generation of diagrams with *FeynArts*

Küblbeck, Eck, Mertig '92, Hahn '01

■ Projection technique:

$$v_f(k^2) = \frac{1}{2(2-d)k^2} \text{Tr}[\gamma_\mu \not{p}_1 z_f^\mu(k^2) \not{p}_2]$$

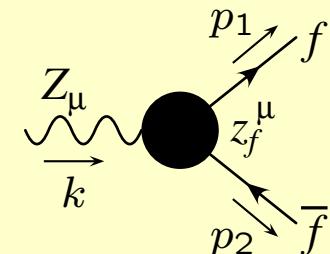
$$a_f(k^2) = \frac{1}{2(2-d)k^2} \text{Tr}[\gamma_5 \gamma_\mu \not{p}_1 z_f^\mu(k^2) \not{p}_2]$$

■ On-shell renormalization:

M_W , M_Z , M_H , m_t defined through (complex) propagator poles

■ Dirac/Lorentz algebra, reduction to basis integrals

■ Evaluation of 2-loop integrals in general not possible analytically



Dispersion relations

Topologies with **self-energy sub-loop** can easily be integrated by using dispersion relation for B_0 function:

S. Bauberger et al. '95

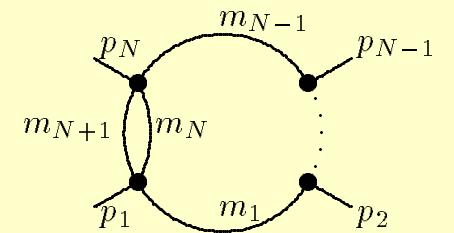
$$B_0(p^2, m_1^2, m_2^2) = - \int_{(m_1+m_2)^2}^{\infty} ds \frac{\Delta B_0(s, m_1^2, m_2^2)}{s - p^2}$$

with $\Delta B_0(s, m_1^2, m_2^2) = (4\pi\mu^2)^{4-D} \frac{\Gamma(D/2 - 1)}{\Gamma(D - 2)} \frac{\lambda^{(D-3)/2}(s, m_1^2, m_2^2)}{s^{D/2-1}}$,

$$\lambda(a, b, c) = (a - b - c)^2 - 4bc$$

$$T_{N+1}(p_i; m_i^2) = - \int_{s_0}^{\infty} ds \Delta B_0(s, m_N^2, m_{N+1}^2)$$

$$\times \int d^4 q \frac{1}{q^2 - s} \frac{1}{(q + p_1)^2 - m_1^2} \cdots \frac{1}{(q + p_1 + \cdots + p_{N-1})^2 - m_{N-1}^2}$$



Feynman parameter integration

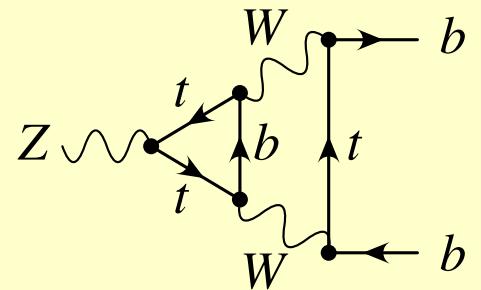
Topologies with **sub-loop triangles**:

Feynman parameter integration

After tensor reduction:

$$I = \int_0^1 dx_1 \dots dx_{n-1} [D_1(A - i\epsilon)^{-1} + D_2(A - i\epsilon)^{-2} + \dots]$$

$D_{1,2,\dots}$, A are polynomials of x_i



Physical thresholds: A changes sign in integration region

→ Problematic for numerical integrators

→ Deform Feynman parameter integration into complex x plane: Nagy, Soper '06

$$x_i = z_i - i\lambda z_i(1 - z_i) \frac{\partial A}{\partial z_i}, \quad 0 \leq z_i \leq 1.$$

$$A(\vec{x}) = A(\vec{z}) - i\lambda \sum_i z_i(1 - z_i) \left(\frac{\partial A}{\partial z_i} \right)^2 + \mathcal{O}(\lambda^2).$$

Typical choice: $\lambda \sim 0.5 - 1$

Feynman parameter integration

UV divergencies: in either or both *subloops*, and also *global*

→ Direct evaluation not possible, need **UV subtraction terms**

Freitas '12

Global UV divergency:

$$G_{\text{glob}} = \frac{N(k_1, k_2)}{D(k_1, k_2)} \Big|_{p_i=0}$$

Works for all two-loop N -point functions except selfenergies

$\int d^d k_1 d^d k_2 G_{\text{glob}}$ consists of two-loop vacuum integrals (known analytically)

Davydychev, Tausk '92

$I_{\text{gs}} \equiv I - \int d^d k_1 d^d k_2 G_{\text{glob}}$ can have singularities in one subloop
(both subloops only for tadpoles and selfenergies)

Feynman parameter integration

Subloop UV divergency in k_1 loop:

- Introduce Feynman parameters for k_1 subloop and perform k_1 tensor reduction (as before)

$$I_{\text{gs}} = \int_0^1 dx_1 \dots dx_{m-1} \int d^d k_1 d^d k_2 \left[\frac{C_1}{[k_1^2 - A]^m} + \frac{C_2}{[k_1^2 - A]^{m-1}} + \dots + \frac{C_j}{[k_1^2 - A]^2} \right],$$

contain all k_2 dependence

of k_1 propagators

UV diverg.

- UV subloop subtraction term:

$$G_{\text{sub}} = \int_0^1 dx_1 \dots dx_{m-1} \frac{C_j}{[k_1^2 - \mu^2]^2},$$

(μ = suitably chosen mass parameter)

Results for Γ_Z

- Checks against existing calculations
for $\sin^2 \theta_{\text{eff}}^\ell, \sin^2 \theta_{\text{eff}}^b, R_b$

Awramik, Czakon, Freitas, Weiglein '04

Awramik, Czakon, Freitas, Kniehl '09

Freitas, Huang '12

- Known corrections to vertex form factors:

$\mathcal{O}(\alpha), \mathcal{O}(\alpha \alpha_s)$

Czarnecki, Kühn '96

Harlander, Seidensticker, Steinhauser '98

$\mathcal{O}(m_t^2 \alpha \alpha_s^2), \mathcal{O}(m_t^4 \alpha^2 \alpha_s), \mathcal{O}(m_t^6 \alpha^3), \mathcal{O}(m_t^2 \alpha \alpha_s^3)$

Avdeev, Fleischer, Mikhailov, Tarasov '94; Chetyrkin, Kühn, Steinhauser '95

Faisst, Kühn, Seidensticker, Veretin '03; Boughezal, Tausk, v. d. Bij '05

Schröder, Steinhauser '05; Chetyrkin et al. '06; Boughezal, Czakon '06

$\mathcal{O}(N_f \alpha^2)$

Freitas '13

- Input parameters:

$M_Z, M_H, m_t, \Delta\alpha, \alpha_s, G_\mu$

For final-state effects: m_b, m_c, m_τ

M_W computed from G_μ

Awramik, Czakon, Freitas, Weiglein '04

Results for Γ_Z

	Γ_Z [GeV]	R_ℓ	R_c	R_b
Born + $\mathcal{O}(\alpha)$	2.49769	20.8031	0.17230	0.21558
$\mathcal{O}(\alpha\alpha_s)$	-0.00121	-0.0068	-0.00008	0.00035
leading 3/4-loop	-0.00089	-0.0020	<0.00001	<0.00001
$\mathcal{O}(N_f^2\alpha^2, N_f\alpha^2)$	-0.00136	-0.0434	0.00001	-0.00013
Total*	2.49430	20.7512	0.17223	0.21580
Large- m_t exp.**	2.49485	20.7484	0.17220	0.21579

* excluding $\mathcal{O}(m_t^2\alpha\alpha_s^3)$

** best known previous result $\mathcal{O}(m_t^4\alpha^2)$, $\mathcal{O}(m_t^2\alpha^2)$ (except for $b\bar{b}$)
 Degrassi, Gambino, Vicini '96; Degrassi, Gambino, Sirlin '97; Degrassi, Gambino '99
 Barbieri et al. '92; Fleischer, Tarasov, Jegerlehner '93,95

$$R_b = \frac{\Gamma[Z \rightarrow b\bar{b}]}{\Gamma[Z \rightarrow \text{had.}]} \quad R_c = \frac{\Gamma[Z \rightarrow c\bar{c}]}{\Gamma[Z \rightarrow \text{had.}]} \quad R_\ell = \frac{\Gamma[Z \rightarrow \text{had.}]}{\Gamma[Z \rightarrow \ell^+\ell^-]}$$

Error estimation for Γ_Z

- Geometric perturbative series

$$\alpha_t = \alpha m_t^2$$

$$\mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha_{\text{ferm}}^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha_{\text{ferm}}^2) \sim 0.26 \text{ MeV}$$

$$\mathcal{O}(\alpha^2 \alpha_s) - \mathcal{O}(\alpha_t^2 \alpha_s) \sim \frac{\mathcal{O}(\alpha_{\text{ferm}}^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha \alpha_s) \sim 0.30 \text{ MeV}$$

$$\mathcal{O}(\alpha \alpha_s^2) - \mathcal{O}(\alpha_t \alpha_s^2) \sim \frac{\mathcal{O}(\alpha \alpha_s) - \mathcal{O}(\alpha_t \alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha \alpha_s) \sim 0.23 \text{ MeV}$$

$$\mathcal{O}(\alpha \alpha_s^3) - \mathcal{O}(\alpha_t \alpha_s^3) \sim \frac{\mathcal{O}(\alpha \alpha_s) - \mathcal{O}(\alpha_t \alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha \alpha_s^2) \sim 0.035 \text{ MeV}$$

$$\mathcal{O}(\alpha_{\text{bos}}^2) \sim \mathcal{O}(\alpha_{\text{bos}})^2 \sim 0.1 \text{ MeV}$$

- Parametric prefactors: $\mathcal{O}(\alpha_{\text{bos}}^2) \sim \Gamma_Z \alpha^2 \sim 0.1 \text{ MeV}$

Total: $\delta \Gamma_Z \approx 0.5 \text{ MeV}$

Outlook

- SM is experimentally confirmed, but no evidence for new physics at collider
→ "Low"-energy precision measurements may lead the way
- Experimental precision from LEP/SLC demands SM prediction with
2-loop corrections and **partial 3-loop corrections**
- Much progress during last 10–20 years:
Complete or *fermionic* 2-loop corrections available for M_W and **all Z-pole observables**; $\Delta_{\text{th}} < \Delta_{\text{exp}}$
- **LHC** will provide independent results for $\sin^2 \theta_{\text{eff}}$ and M_W ,
but overall precision not improved
- **ILC/TLEP** with $\sqrt{s} \sim M_Z$ will reduce experimental error by at least $\mathcal{O}(2 – 10)$
→ Challenge for theorists!

Backup slides

Z-pole cross-section

Peak cross section:

$$\sigma_{\text{had}}^0 = \frac{1}{64\pi^2 s} \sum_q \int d\Omega \left| \mathcal{A}(e^+ e^- \rightarrow q\bar{q}) \right|_{s=M_Z^2}^2$$

Explicit calculation:

Freitas '13

$$\sigma_{\text{had}}^0 = \frac{12\pi}{M_Z^2} \sum_q \frac{\Gamma_e \Gamma_f}{\Gamma_Z^2} (1 + \delta X)$$

Correction term first at NNLO:

Grassi, Kniehl, Sirlin '01

$$\delta X_{(2)} = -(\text{Im } \Sigma'_{Z(1)})^2 - 2\Gamma_Z M_Z \text{ Im } \Sigma''_{Z(1)}$$

Bulk of electroweak corrections for Γ_Z and σ_{had}^0 contained in g_V^f, g_A^f, Σ'_Z ;

known to $\mathcal{O}(\alpha\alpha_s), \mathcal{O}(N_f\alpha^2)$

Czarnecki, Kühn '96

Harlander, Seidensticker, Steinhauser '98; Freitas '13

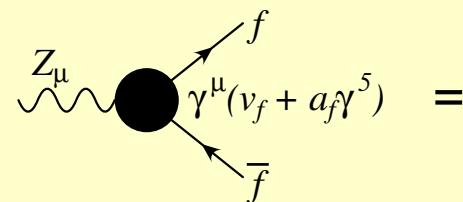
and partial 3/4-loop

Z - γ mixing

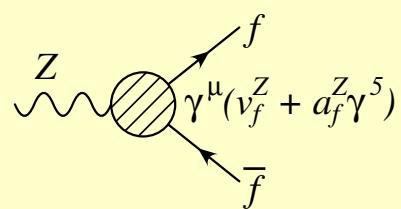
$$v_f(s) = v_f^Z(s) - v_f^\gamma(s) \frac{\Sigma_{\gamma Z}(s)}{s + \Sigma_{\gamma\gamma}(s)},$$

(similar for a_f)

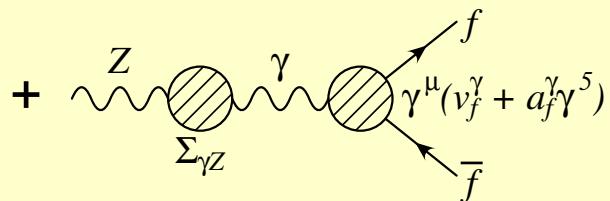
$$\Sigma_Z(s) = \Sigma_{ZZ}(s) - \frac{[\Sigma_{\gamma Z}(s)]^2}{s + \Sigma_{\gamma\gamma}(s)},$$



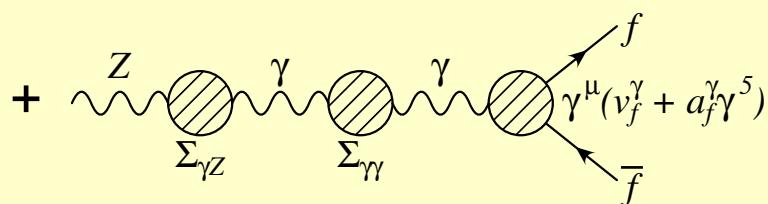
$$= \Sigma_Z$$



$$= \Sigma_{ZZ}$$



$$+ \Sigma_{\gamma Z} \Sigma_{\gamma Z}$$

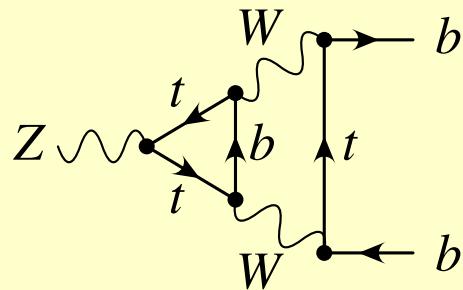


$$+ \Sigma_{\gamma Z} \Sigma_{\gamma\gamma} \Sigma_{\gamma Z}$$

+ ...

+ ...

γ_5



$$\{\gamma^\mu, \gamma_5\} = 0$$

$$\text{In 4 dim.: } \text{tr}\{\gamma^\alpha \gamma^\beta \gamma^\gamma \gamma^\delta \gamma_5\} = 4i \epsilon^{\alpha\beta\gamma\delta}$$

$$\text{In } d \text{ dim.: } \text{tr}\{\gamma^\alpha \gamma^\beta \gamma^\gamma \gamma^\delta \gamma_5\} = 0$$

Contributions with ϵ -tensor are UV-finite and gauge-invariant

Adler, Bardeen '69

→ Compute this part in 4 dim.

Jones, Leveille '82

Jegerlehner '00

Use photon mass to avoid (spurious) IR divergencies

Awramik, Czakon, Freitas '06

Reduction to master integrals

- Passarino-Veltman reduction (for sub-loop bubbles)

Weiglein, Scharf, Böhm '94

- Integration by parts & Lorentz-invariance identities

Chetyrkin, Tkachov '81

Gehrman, Remiddi '00

- Derivative of 2-loop self-energy:

$$T(p^2; m_1^2, m_2^2, m_3^2, m_4^2, m_5^2; \nu_1, \nu_2, \nu_3, \nu_4, \nu_5) = -\frac{(4\pi^2 \mu^2)^{4-d}}{\pi^4} \\ \times \int \frac{d^d q_1 d^d q_2}{[q_1^2 - m_1^2]^{\nu_1} [(q_1 + p)^2 - m_2^2]^{\nu_2} [(q_1 - q_2)^2 - m_3^2]^{\nu_3} [q_2^2 - m_4^2]^{\nu_4} [(q_2 + p)^2 - m_5^2]^{\nu_5}}$$

$$\frac{\partial T}{\partial(p^2)} = \frac{1}{2p^2} p^\mu \frac{\partial T}{\partial p^\mu} = -\frac{1}{2p^2} \left[(\nu_2 + \nu_5)T - \nu_2 T(\nu_1 - 1, \nu_2 + 1) - \nu_5 T(\nu_4 - 1, \nu_5 + 1) \right. \\ \left. + (m_2^2 - m_1^2 + p^2)T(\nu_2 + 1) + (m_5^2 - m_4^2 + p^2)T(\nu_5 + 1) \right].$$

→ Same class of functions, can then be reduced with i.b.p. identities

Two-loop UV divergencies

Subloop UV divergency in k_1 loop:

- Integrated subtraction term:

$$\int d^d k_1 d^d k_2 G_{\text{sub}} = -\Gamma(\varepsilon - 2) \mu^{2-\varepsilon} \int_0^1 dy_1 \dots dy_{m-1} \underbrace{\int d^d k_2 C_j}_{\text{one-loop integral}}.$$

(same procedure as above)

- Remainder $I_{\text{rem}} \equiv I - \int d^d k_1 d^d k_2 G_{\text{glob}} - \int d^d k_1 d^d k_2 G_{\text{sub}}$ is finite
- Feynman parameters for k_2 subloop
- Tensor reduction for k_2 terms
- Perform k_1 and k_2 integrals
 - Numerical integral over Feynman parameters
- Deform integration contour as necessary

Future challenges

Precision at proposed future colliders:

	ILC	TLEP	Th. error with 3-loop [†]	Param. error ILC*	Param. error TLEP**
M_W [MeV]	3–5	~ 1	1	2.6	1
Γ_Z [MeV]	~ 1	$\lesssim 1$	$\lesssim 0.2$	0.5	?
R_b [10^{-5}]	15	$\lesssim 5$	5–10	< 1	< 1
$\sin^2 \theta_{\text{eff}}^\ell$ [10^{-5}]	1.3	0.3	1.5	2	2

[†] Theory improvements: $\mathcal{O}(N_f \alpha^2 \alpha_s)$, $\mathcal{O}(N_f^3 \alpha^3)$, $\mathcal{O}(N_f^2 \alpha^3)$

Parametric inputs:

* **ILC:** $\delta m_t = 100$ MeV, $\delta \alpha_s = 0.001$, $\delta M_Z = 2.1$ MeV

****TLEP:** $\delta m_t \lesssim 50$ MeV, $\delta \alpha_s = ?$, $\delta M_Z = 0.1$ MeV

also: $\delta(\Delta \alpha) = 5 \times 10^{-5}$

Electroweak precision tests: new physics

Constraints on Higgs physics:

- Indirect determination of M_H
- Couplings: need to go beyond SM, e.g. THDM:

$$\left| \frac{g_{hVV}^{\text{THDM}}}{g_{hVV}^{\text{SM}}} \right| = \sin(\beta - \alpha),$$

$$\left| \frac{g_{hff}^{\text{THDM}}}{g_{hff}^{\text{SM}}} \right| = \frac{\cos \alpha}{\sin \alpha} \quad \text{or} \quad \frac{\sin \alpha}{\cos \alpha}$$

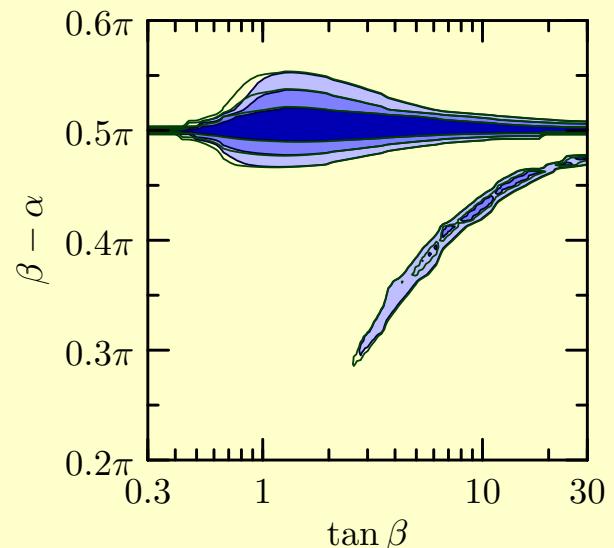
- Couplings: effective operators

$$\mathcal{O}_W = (D^\mu \Phi)^\dagger W_{\mu\nu} (D^\nu \Phi)$$

$$\mathcal{O}_{WW} = \Phi^\dagger W^{\mu\nu} W_{\mu\nu} \Phi$$

etc.

Mebane, Greiner, Zhang, Willenbrock '13
Chen, Dawson, Zhang '13



Eberhardt, Nierste, Wiebusch '13

